Calculus Warm Up #5-4

1. Differentiate using the quotient rule. Simplify result.

$$f(x) = \frac{x^2}{x^2 + 3}$$

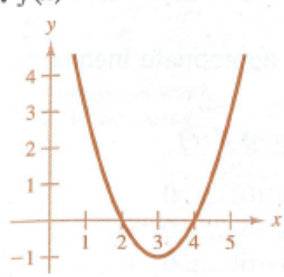
2. Find, without a grapher, the zeros and asymptotes for the graph of

$$f(x) = \frac{x^3 + 5x^2 + 2x - 8}{x^2 - 2}$$

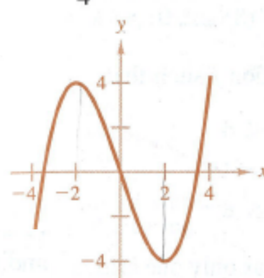
HW Questions: p. 173

In Exercises 1–6, identify the open intervals on which the function is increasing or decreasing.

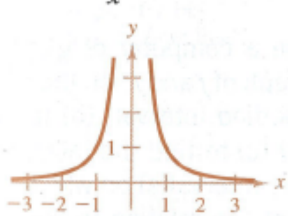
1. $f(x) = x^2 - 6x + 8$



3. $y = \frac{x^3}{4} - 3x$



5. $f(x) = \frac{1}{x^2}$



In Exercises 7–18, find the critical numbers of f (if any), find the open intervals on which f is increasing or decreasing, and locate all relative extrema.

7. $f(x) = -2x^2 + 4x + 3$
9. $f(x) = x^2 - 6x$
11. $f(x) = 2x^3 + 3x^2 - 12x$
13. $f(x) = x^{1/3} + 1$
15. $f(x) = \dfrac{x^2}{x^2 - 9}$ $\rightarrow$ $f'(x) = \dfrac{4x^3 - 18x}{x - 3}$
17. $f(x) = \dfrac{x^5 - 5x}{5}$

In Exercises 19–26, find the critical numbers of f (if any), find the open intervals on which the algebraic function is increasing or decreasing, and locate all relative extrema. Use a graphing utility to confirm your results.

19. $f(x) = x^3 - 6x^2 + 15$
21. $f(x) = (x - 1)^{2/3}$

In Exercises 19–26, find the critical numbers of f (if any), find the open intervals on which the algebraic function is increasing or decreasing, and locate all relative extrema. Use a graphing utility to confirm your results.

23. $f(x) = x + \dfrac{1}{x}$

$f(x) = \dfrac{x^2+1}{x}$

$f'(x) = \dfrac{x(2x) - (x^2+1)(1)}{x^2}$

$f'(x) = \dfrac{x^2-1}{x^2}$

critical #'s $f'(x) = 0$ @ $x = \pm 1$

$f'(x)$ is undefined @ $x = 0$

f' test: -2 $\quad -\frac{1}{2}$ $\quad \frac{1}{2}$ $\quad 2$

$+ \quad -1 \quad - \quad 0 \quad - \quad 1 \quad +$

Max: $(-1,-2)$ Min: $(1,2)$

f is increasing: $(-\infty,-1) \cup (1,\infty)$

f is decreasing: $(-1,0) \cup (0,1)$

25. $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$

In Exercises 27 and 28, determine whether the given function is strictly monotonic on the indicated interval.

27. $f(x) = x^2$

(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$ (c) $(0, \infty)$

29. The height (in feet) of a ball at time t (in seconds) is given by the position function

$$s(t) = 96t - 16t^2.$$

Find the open interval on which the ball is moving up and the open interval on which it is moving down. What is the maximum height of the ball?

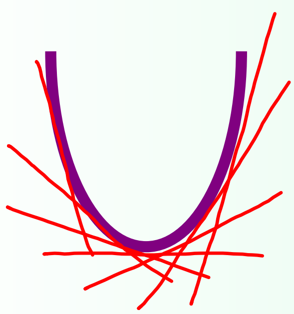
4.4

-Concavity

-Points of inflection

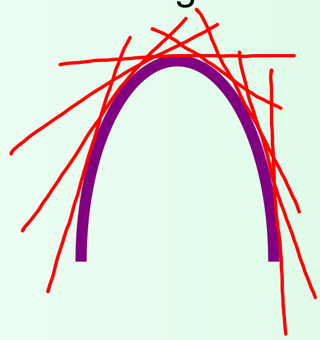
-The Second Derivative Test

Concavity: Read graphs from left to right.



Concave up:

Slopes, f´(x),
are increasing.



Concave down:

Slopes, f´(x),
are decreasing.

Yesterday we used the first derivative to determine where the function was increasing or decreasing.

Where $f'(x)$ is + , f is increasing.

Where $f'(x)$ is - , f is decreasing.

We can use the second derivative to determine where the slopes are increasing or decreasing.

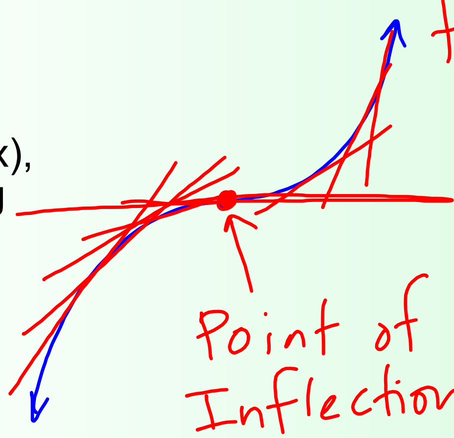
Where $f''(x)$ is + , $f'(x)$ is increasing. f(x) is concave up.

Where $f''(x)$ is - , $f'(x)$ is decreasing. f(x) is concave down.

Slopes changing from increasing to deceasing indicates a change in concavity.

$f''(x)$ is +

Slopes, $f'(x)$, decreasing



f(x) is: concave up.

$f''(x)$ is −

f(x) is: concave down

Point of Inflection

$f''(x) = 0$

EXAMPLE 1 Determining concavity

Determine the open intervals on which the graph of $f(x) = 6(x^2 + 3)^{-1}$ is concave upward or downward.

Plan: Find where $f''(x) = 0$ or undefined. Test the intervals between.

$f'(x) = -6(x^2+3)^{-2}(2x)$

$f'(x) = -12x(x^2+3)^{-2}$

$f''(x) = -12x(-2)(x^2+3)^{-3}(2x) + (-12)(x^2+3)^{-2}$

$= \dfrac{48x^2}{(x^2+3)^3} - \dfrac{12}{(x^2+3)^2} \cdot \dfrac{(x^2+3)}{(x^2+3)}$

$0 = \dfrac{36x^2 - 36}{(x^2+3)^3}$

$f''(x)$ is not undef. anywhere.

$0 = 36(x^2-1)$

$x = \pm 1$

test: -2, 0, 2

-2	-1	0	1	2
$\frac{+}{+}$ = (+)		$\frac{-}{+}$ = (−)		$\frac{+}{+}$ = (+)

concave up: $(-\infty, -1) \cup (1, \infty)$

concave down: $(-1, 1)$

We can use the second derivative to test #'s in between x-values of interest to determine concavity of a function.

This is different than what is called:

The Second Derivative Test

THEOREM 4.9 SECOND DERIVATIVE TEST for Extrema

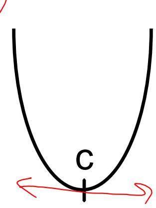
Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a relative minimum.
2. If $f''(c) < 0$, then $f(c)$ is a relative maximum.
3. If $f''(c) = 0$, then the test fails.

Given $f'(c) = 0$

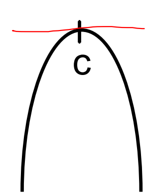
If $f''(c) > 0$

Min @ $x = c$



If $f''(c) < 0$

Max @ $x = c$



EXAMPLE 4 Using the Second Derivative Test

Find the relative extrema for $f(x) = -3x^5 + 5x^3$.

$f'(x) = -15x^4 + 15x^2$

$0 = -15x^2(x^2 - 1)$

$x = 0, \pm 1$

2nd Deriv. Test

$f''(x) = -60x^3 + 30x$

$f''(-1) = 60 - 30 = 30$ $+ \rightarrow$ Min @ $(-1,-2)$

$f''(0) = 0$ ☹ test for extrema failed.

$f''(1) = -60 + 30 = -30$ $- \rightarrow$ Max @ $(1,2)$

Graph sketching:

1. Find the intercepts, asymptotes and check end behavior for what to expect. (Use your Precalc tools!)

2. Use the first derivative and critical numbers to find where the graph is increasing, decreasing or has possible extrema.

3. Use the second derivative to determine shape (concavity).

Graph Sketching Classwork: (No Calculator)

Organize all the information about the function and use it to accurately sketch the graph by hand.

$f(x) = x^4 - 4x^3$

Intercepts:

Asymptotes:

End behavior:

$f'(x)$:

Critical #'s:

$f''(x)$:

Graph sketching:

1. Find the intercepts, asymptotes and check end behavior for what to expect. (Use your Precalc tools!)
2. Use the first derivative and critical numbers to find where the graph is increasing, decreasing or has possible extrema.
3. Use the second derivative to determine shape (concavity).

Table for investigating the function:

Places of interest include x-values where the first or second derivative = 0 or is undefined.

$x = 0$
$x = 2$
$x = 3$

Conclusion column is for extrema, points of inflection, function increasing or decreasing, and concavity

Places of interest and intervals between	test #'s on the interval	$f'(x)$ +, - or 0	$f''(x)$ +, - or 0	Conclusion
$(-\infty, 0)$				
$x = 0$				
$(0, 2)$				
$x = 2$				
$(2, 3)$				
$x = 3$				
$(3, \infty)$				

HW:

p. 180 # 3 - 11 odd,
15, 23, 25, 29